



ARJUNA JEE 2025

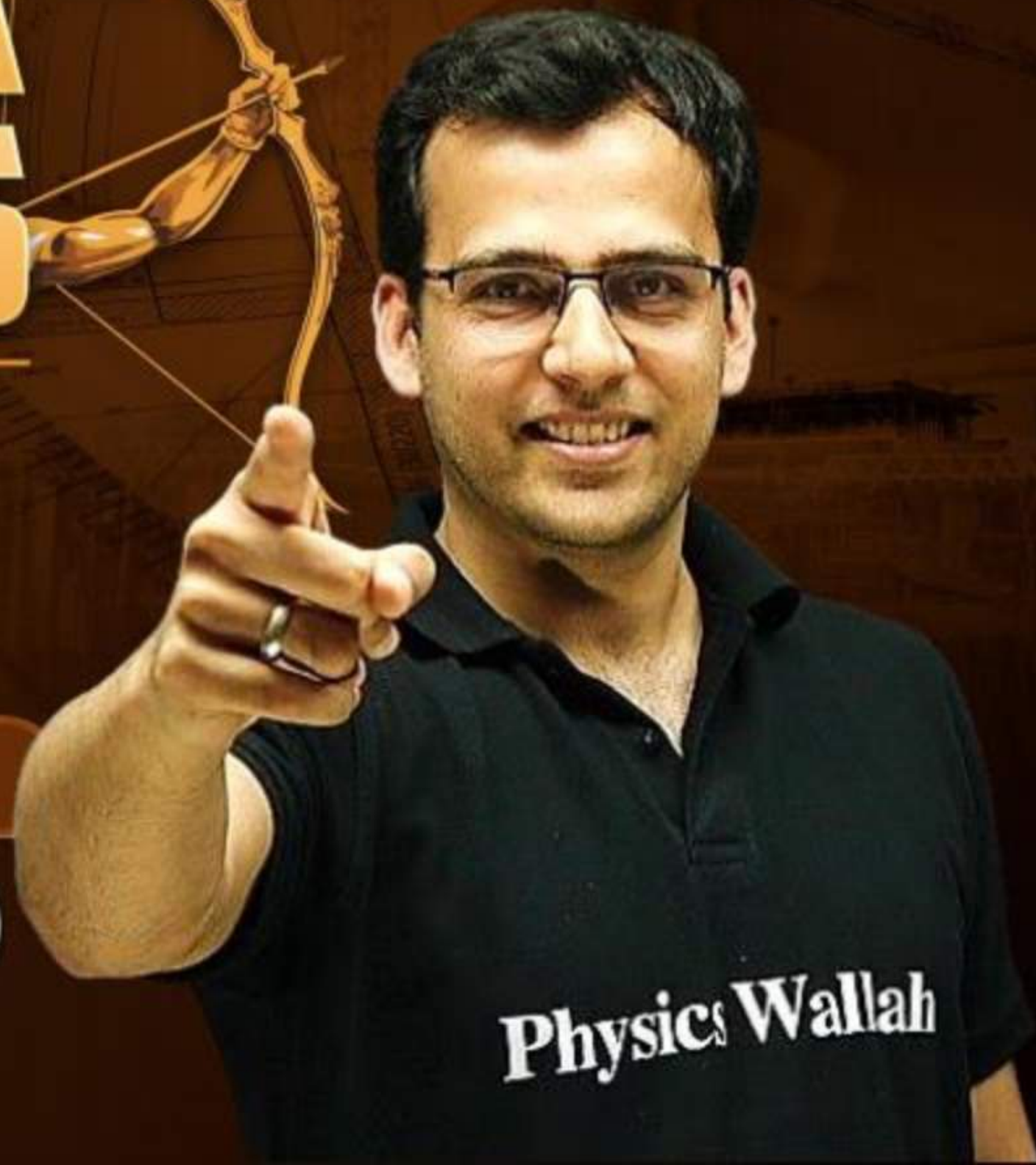
Trigonometric Identities

Mathematics

Lecture – 01

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Today's **Targets**

1

Revision of Class 10th Formulas

2

Sign Convention in Trigonometry

3

ASTC Rule



Class 10th RECAP



Trigonometric ratios for θ are defined as below:

(i) $\text{sine } \theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{P}{H}$ and is written as $\sin \theta$.

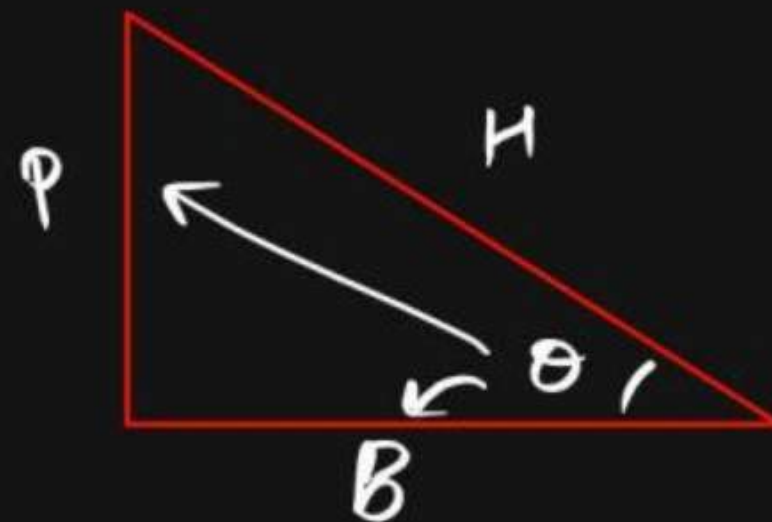
(ii) $\text{cosine } \theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{B}{H}$ and is written as $\cos \theta$.

(iii) $\text{tangent } \theta = \frac{\text{perpendicular}}{\text{base}} = \frac{P}{B}$ and is written as $\tan \theta$.

(iv) cosecant $\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{H}{P}$ and is written as cosec θ .
Cosec θ

(v) secant $\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{H}{B}$ and is written as sec θ .

(vi) cotangent $\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{B}{P}$ and is written as cot θ .



$$\text{cosec } \theta = \frac{1}{\sin \theta}$$

$$\text{sec } \theta = \frac{1}{\cos \theta}$$

$$\text{cot } \theta = \frac{1}{\tan \theta}$$



Class 10th RECAP



Values of all the Trigonometric Ratios of 0° , 30° , 45° , 60° and 90°

θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D.
$\operatorname{cosec} \theta$	N.D	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	N.D
$\cot \theta$	N.D	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0



Class 10th RECAP



(i) $\sin^2 \theta + \cos^2 \theta = 1;$

(ii) $1 + \tan^2 \theta = \sec^2 \theta;$

(iii) $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta.$

$$\sin^2 \theta = 1 - \cos^2 \theta = (1 - \cos \theta)(1 + \cos \theta)$$

$$\text{OR } \cos^2 \theta = 1 - \sin^2 \theta = (1 - \sin \theta)(1 + \sin \theta)$$

$$\Rightarrow \boxed{\sec^2 \theta - \tan^2 \theta = 1} \Rightarrow (\sec \theta - \tan \theta)(\sec \theta + \tan \theta) = 1$$

$$** \boxed{\sec \theta + \tan \theta = \frac{1}{\sec \theta - \tan \theta}}$$

$$\Rightarrow \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$(\operatorname{cosec} \theta - \cot \theta)(\operatorname{cosec} \theta + \cot \theta) = 1$$

$$\boxed{\operatorname{cosec} \theta + \cot \theta = \frac{1}{\operatorname{cosec} \theta - \cot \theta}}$$

$$\boxed{\sin^2 \theta + \cos^2 \theta = 1}$$

$$\Rightarrow \sin^4 \theta + \cos^4 \theta = 1 \text{ (WRONG)}$$

$$\rightarrow \text{S.B.S.} \Rightarrow (\sin^2 \theta + \cos^2 \theta)^2 = 1 \Rightarrow (\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta = 1$$

$$\# \boxed{\sin^4 \theta + \cos^4 \theta = 1 - 2 \sin^2 \theta \cos^2 \theta} \quad \sin^4 \theta + \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta = 1$$

Question



Prove that, $\sin^3 A + \cos^3 A = (\sin A + \cos A)(1 - \sin A \cos A)$

LHS:

$$(\sin A + \cos A) \left(\underbrace{\sin^2 A + \cos^2 A}_1 - \sin A \cos A \right)$$

$$(\sin A + \cos A) (1 - \sin A \cos A)$$

RHS

#

$$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

$$a \rightarrow \sin A$$

$$b \rightarrow \cos A$$

$$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$$

Question

Prove that, $\cos^6 A + \sin^6 A = 1 - 3\sin^2 A \cos^2 A$

LHS:

$$(\cos^2 A)^3 + (\sin^2 A)^3$$

$$\underbrace{(\cos^2 A + \sin^2 A)}_1 \underbrace{(\cos^4 A + \sin^4 A - \cos^2 A \sin^2 A)}_{\downarrow}$$

$$\left[1 - \underbrace{2\sin^2 A \cos^2 A - \cos^2 A \sin^2 A} \right]$$

$$\left[1 - 3\sin^2 A \cos^2 A \right]$$

RHS

$$a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$$

$$a \rightarrow \cos^2 A$$

$$b \rightarrow \sin^2 A$$

$$\cos^6 A = (\cos^2 A)^3$$

Question



Prove that , $\sin^8 A - \cos^8 A = (\sin^2 A - \cos^2 A)(1 - 2\sin^2 A \cos^2 A)$

$$\text{LHS: } (\sin^4 A)^2 - (\cos^4 A)^2$$

$$(\sin^4 A + \cos^4 A)(\sin^4 A - \cos^4 A)$$

$$(1 - 2\sin^2 A \cos^2 A) [(\sin^2 A)^2 - (\cos^2 A)^2]$$

$$(1 - 2\sin^2 A \cos^2 A) (\sin^2 A - \cos^2 A)(\sin^2 A + \cos^2 A)$$

RHS



RECAP



Trigonometric Ratios of Complementary Angles

θ & $90^\circ - \theta$ are complementary

Ex $\rightarrow 20^\circ$ & 70°

Ex $\rightarrow 10^\circ$ & 80°

Ex $\rightarrow 15^\circ$ & 75°

$$\sin 10^\circ = \cos 80^\circ$$

$$\sin 40^\circ = \cos 50^\circ$$

$$\cos 72^\circ = \sin 18^\circ$$

$$\cos 1^\circ = \sin 89^\circ$$

$$1) \sin(90^\circ - \theta) = \cos \theta$$

$$2) \cos(90^\circ - \theta) = \sin \theta$$

$$3) \tan(90^\circ - \theta) = \cot \theta$$

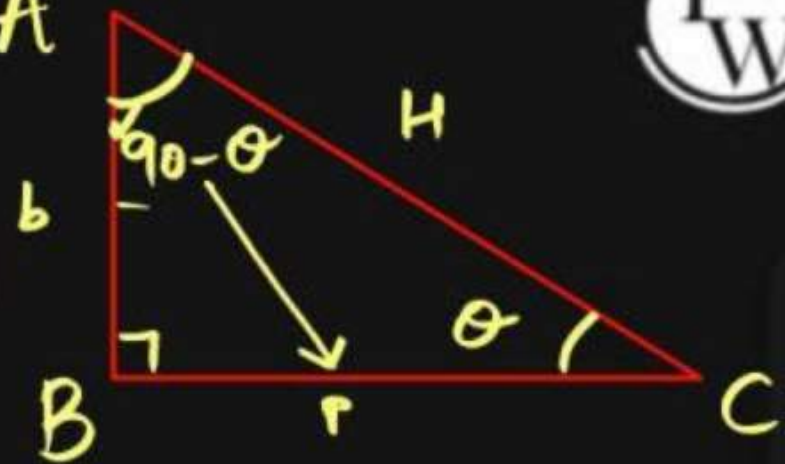
$$4) \cot(90^\circ - \theta) = \tan \theta$$

$$5) \sec(90^\circ - \theta) = \operatorname{cosec} \theta$$

$$6) \operatorname{cosec}(90^\circ - \theta) = \sec \theta$$

Ex: $\sec 30^\circ = \operatorname{cosec} 60^\circ$

Proof $\rightarrow$



$$\sin \theta = \frac{AB}{AC} \sim \textcircled{1}$$

$$\cos(90^\circ - \theta) = \frac{AB}{AC} \sim \textcircled{2}$$

$$\textcircled{1} = \textcircled{2}$$

$$\boxed{\sin \theta = \cos(90^\circ - \theta)}$$

✓

Question



$$2 \left(\frac{\cos^2 20^\circ + \cos^2 70^\circ}{\sin^2 25^\circ + \sin^2 65^\circ} \right) - \tan 45^\circ + \tan 13^\circ \tan 23^\circ \tan 30^\circ \tan 67^\circ \tan 77^\circ$$

Annotations: $\sin^2 20^\circ$ points to $\cos^2 70^\circ$. $\cos^2 25^\circ$ points to $\sin^2 65^\circ$. $\tan 45^\circ$ is boxed with a '1' below it. $\tan 13^\circ$ and $\tan 77^\circ$ are circled. $\tan 67^\circ$ has an arrow pointing to $\cot 23^\circ$. $\tan 30^\circ$ has an arrow pointing to $\frac{1}{\sqrt{3}}$.

$$2 \left[\frac{\cos^2 20^\circ + \sin^2 20^\circ}{\sin^2 25^\circ + \cos^2 25^\circ} \right]$$

$$\frac{2 \times 1}{1} = 2$$

$$\text{Ans } 2 - 1 + \frac{1}{\sqrt{3}} = 1 + \frac{1}{\sqrt{3}} \quad \underline{\text{Ans}}$$

$$\tan \theta \cot \theta = 1$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\begin{aligned} \cos 70^\circ &= \sin 20^\circ \\ \cos^2 70^\circ &= \sin^2 20^\circ \\ \sin^2 65^\circ &= \cos^2 25^\circ \\ \tan 77^\circ &= \cot 13^\circ \\ \tan 30^\circ &= \frac{1}{\sqrt{3}} \end{aligned}$$

Question



$$\left(\frac{\tan 20^\circ}{\operatorname{cosec} 70^\circ}\right)^2 + \left(\frac{\cot 20^\circ}{\sec 70^\circ}\right)^2 + 2 \tan 15^\circ \tan 37^\circ \tan 53^\circ \tan 60^\circ \tan 75^\circ$$

$$\left(\frac{\tan 20}{\sec 20}\right)^2 + \left(\frac{\cot 20}{\operatorname{cosec} 20}\right)^2$$

$$\left(\frac{\sin 20}{\cancel{\cos 20} \sec 20}\right)^2 + \left(\frac{\cos 20}{\cancel{\sin 20} \operatorname{cosec} 20}\right)^2$$

$$\sin^2 20^\circ + \cos^2 20^\circ$$

$$= 1$$

$$\text{Ans} = 1 + 2\sqrt{3}$$

$\sqrt{3}$

$\cot 15^\circ$

$$\operatorname{cosec} 70^\circ = \sec 20^\circ$$

$$\sec 70^\circ = \operatorname{cosec} 20^\circ$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \dots \tan 89^\circ = 1$$

$$\cancel{\tan 44^\circ} \tan 45^\circ \cancel{\tan 46^\circ}$$

$$\cancel{\tan 1^\circ} \cancel{\tan 2^\circ} \tan 3^\circ \tan 4^\circ \dots \dots \dots$$

$$\underbrace{\tan 88^\circ}_{\cot 2^\circ} \underbrace{\tan 89^\circ}_{\cot 1^\circ} = ?$$

$$= \tan 45^\circ$$

$$= 1$$

Question



$$\sin^2 5^\circ + \sin^2 10^\circ + \dots + \sin^2 85^\circ + \sin^2 90^\circ = 9\frac{1}{2}$$

Annotations: A bracket above the first three terms is labeled (1). A bracket below the last three terms is labeled $\cos^2 5^\circ$. A bracket below the last two terms is labeled 1. A bracket below the entire sum is labeled (1).

$$\sin^2 85^\circ = \cos^2 5^\circ$$

$$\sin^2 80^\circ = \cos^2 10^\circ$$

$$\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \dots + \sin^2 40^\circ + \sin^2 45^\circ + \sin^2 50^\circ + \dots + \sin^2 85^\circ + \sin^2 90^\circ$$

Annotations: A bracket above the first four terms is labeled (1). A bracket below the last four terms is labeled $\cos^2 40^\circ$. A bracket below the entire sum is labeled (1).

$$8 + \sin^2 45^\circ + \sin^2 90^\circ$$

$$8 + \left(\frac{1}{\sqrt{2}}\right)^2 + (1)^2$$

$$= 9 + \frac{1}{2} = 9.5 \text{ Ans}$$

Question



If θ is an acute angle such that $5\cot \theta = 3$, then find the value of $\frac{5\sin \theta - 3\cos \theta}{4\sin \theta + 3\cos \theta}$

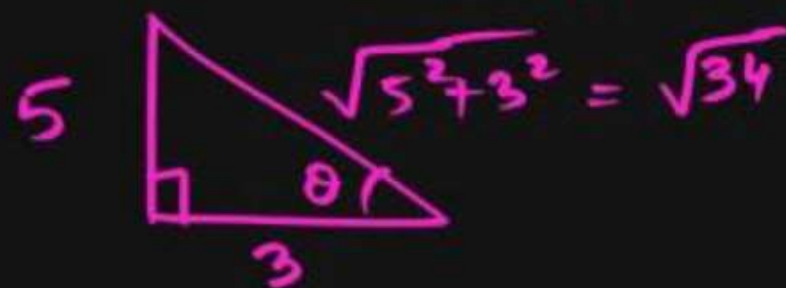
A $\frac{11}{18}$

B $\frac{16}{29}$

C $\frac{14}{27}$

D none of these

$$\cot \theta = \frac{3}{5} = \frac{b}{p}$$



Method $\rightarrow 2$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

divide N^r & D^r with $\sin \theta$

$$\frac{5 - 3 \frac{\cos \theta}{\sin \theta}}{4 + 3 \frac{\cos \theta}{\sin \theta}}$$

$$\frac{5 - 3 \cot \theta}{4 + 3 \cot \theta}$$

$$= \frac{5 - 3 \times \frac{3}{5}}{4 + 3 \times \frac{3}{5}} = \frac{5 - \frac{9}{5}}{4 + \frac{9}{5}} = \frac{\frac{25-9}{5}}{\frac{20+9}{5}} = \frac{16}{29}$$

Question



$$(\operatorname{cosec}^2 72^\circ - \tan^2 18^\circ) =$$

- A 0 $\rightarrow \sec^2 18^\circ - \tan^2 18^\circ$
- B 1 ✓ $= 1$
- C $\frac{3}{2}$
- D none of these

Q.

[Main Jan. 11, 2019 (I)]

Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for $k = 1, 2, 3, \dots$. Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to:

A

$$\frac{1}{12}$$

B

$$\frac{1}{4}$$

C

$$-\frac{1}{12}$$

D

$$\frac{5}{12}$$

$$f_4(x) = \frac{1}{4} [\sin^4 x + \cos^4 x] = \frac{1}{4} [1 - 2 \sin^2 x \cos^2 x] = \frac{1}{4} - \frac{1}{2} \sin^2 x \cos^2 x$$

$$f_6(x) = \frac{1}{6} [\sin^6 x + \cos^6 x] = \frac{1}{6} [1 - 3 \sin^2 x \cos^2 x] = \frac{1}{6} - \frac{1}{2} \sin^2 x \cos^2 x$$

$$\begin{aligned} & \textcircled{1} - \textcircled{2} \\ & \left(\frac{1}{4} - \frac{1}{2} \sin^2 x \cos^2 x \right) - \left(\frac{1}{6} - \frac{1}{2} \sin^2 x \cos^2 x \right) \\ & = \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12} = \frac{1}{12} \end{aligned}$$

Question

If $\sec \theta + \tan \theta = p$, then $\cos \theta =$

A $\frac{2p}{1-p^2}$

B $\frac{2p}{1+p^2}$ ✓

C $\frac{2p}{p^2-1}$

D $\frac{p}{p^2+1}$

If $\sec \theta + \tan \theta = p \sim \textcircled{1}$

$\sec \theta - \tan \theta = \frac{1}{p} \sim \textcircled{2}$

$\textcircled{+}$

$$2 \sec \theta = p + \frac{1}{p}$$

$$2 \sec \theta = \frac{p^2 + 1}{p}$$

$$\sec \theta = \frac{p^2 + 1}{2p}$$

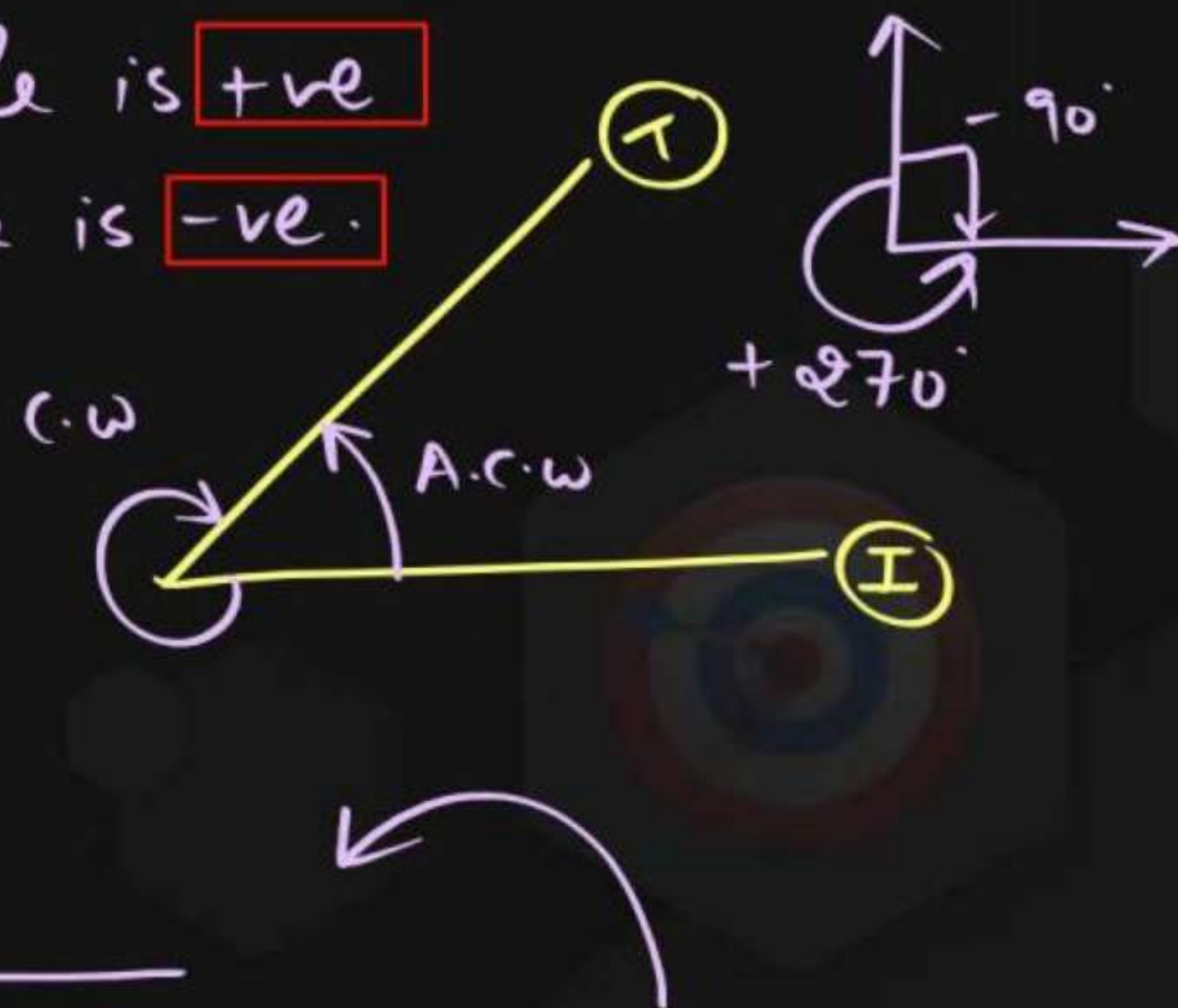
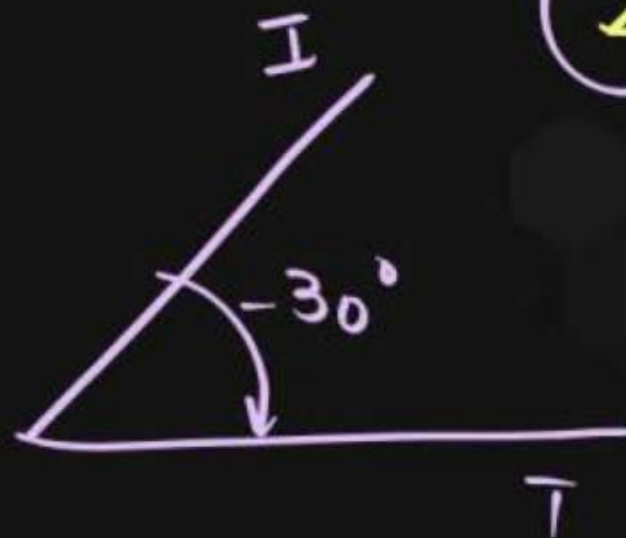
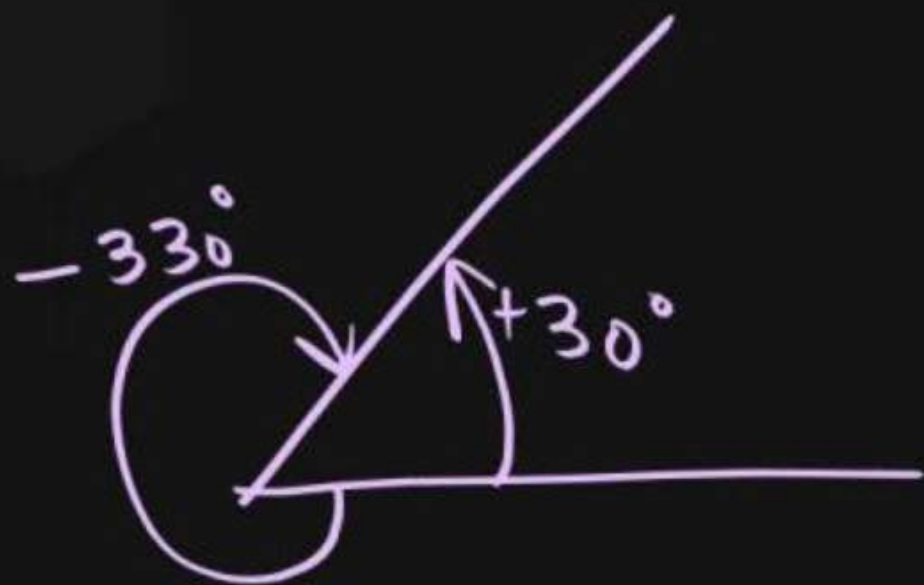
$$\cos \theta = \frac{2p}{p^2 + 1}$$



Sign Convention in Trigonometry

1) In trigonometry angles are always measured from Initial Side to terminal side

2) If rotation is **A.C.W** then angle is **+ve**
If rotation is **C.W** then angle is **-ve**.





ASTC RULE



$$\begin{array}{r} 850 \\ 720 \\ \hline 130 \\ 2000 \\ - 1800 \\ \hline 200 \end{array}$$

$$280^\circ \rightarrow \text{IV}^{\text{th}}$$

$$181^\circ \rightarrow \text{II}$$

$$95^\circ \rightarrow \text{II}^{\text{nd}}$$

$$40^\circ \rightarrow \text{I}^{\text{st}}$$

$$1) \cos 190^\circ < 0$$

$$2) \tan 95^\circ < 0$$

$$3) \cot 195^\circ > 0$$

$$4) \sec 330^\circ > 0$$

$$5) \csc 330^\circ < 0$$

$$6) \sin 170^\circ > 0$$

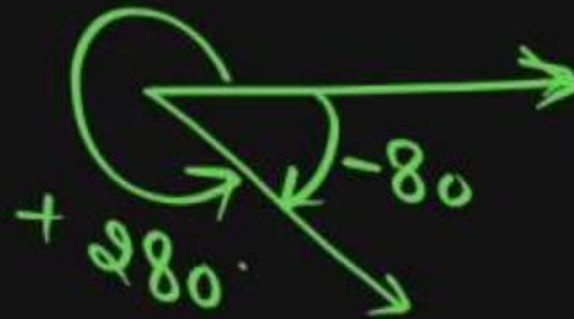
$$7) \sin (800^\circ) = \sin 80^\circ > 0$$

$$8) \cos (850^\circ) = \cos 130^\circ < 0$$

$$9) \tan 2000^\circ = \tan 200^\circ > 0$$

$$+ 390^\circ$$

$$400^\circ - 360^\circ = \textcircled{40^\circ}$$



$$\begin{array}{r} 1000^\circ \\ - 720^\circ \\ \hline 280^\circ \\ - 360^\circ \\ \hline - 80^\circ \\ \hline \hline \end{array}$$





Homework

Prove that

$$\cos^4 A - \sin^4 A + 1 = 2\cos^2 A$$

$$(\sin A + \cos A)(1 - \sin A \cos A) = \sin^3 A + \cos^3 A$$

$$\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} = 2\operatorname{cosec} A$$

$$\cos^6 A + \sin^6 A = 1 - 3\sin^2 A \cos^2 A$$

$$\sqrt{\frac{1 - \sin A}{1 + \sin A}} = \sec A - \tan A$$

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$$



Homework

Prove that

$$\frac{\operatorname{cosec} A}{\cot A + \tan A} = \cos A$$

$$\frac{1}{\cot A + \tan A} = \sin A \cos A$$

$$\frac{1}{\sec A - \tan A} = \sec A + \tan A$$

$$\frac{1 - \tan A}{1 + \tan A} = \frac{\cot A - 1}{\cot A + 1}$$

$$\frac{\sec A - \tan A}{\sec A + \tan A} = 1 - 2 \sec A \tan A + 2 \tan^2 A$$

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$$



Homework



Class 10th Revise



THANK YOU



Lage Raho !!